

1. The  $n$ th even number is  $2n$ .

The next even number after  $2n$  is  $2n + 2$

(a) Explain why.

.....  
 .....

(1)

(b) Write down an expression, in terms of  $n$ , for the next even number after  $2n + 2$

.....

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

(3)  
 (Total 5 marks)

2. Tarish says,  
‘The sum of two prime numbers is always an even number’.

He is **wrong**.  
Explain why.

.....  
.....

(Total 2 marks)

3. Prove that  $(3n + 1)^2 - (3n - 1)^2$  is a multiple of 4, for all positive integer values of  $n$ .

(Total 3 marks)

- |    |              |   |   |
|----|--------------|---|---|
| 1. | (a) Add on 2 | <i>BI 'even numbers go up in twos' or 'even numbers are 2 apart'</i><br><i>oe</i> | 1 |
|    | (b) $2n + 4$ | <i>BI <math>2n + 4</math> oe</i>  | 1 |

(c)  $2n + 2n + 2 + 2n + 4 = 6n + 6$   
 $= 6(n + 1)$

3

*M1 for  $2n (+) 2n + 2 (+) '2n + 4'$  or any 3 consecutive even numbers written as expressions; any variable may be used*

*A1 for " $6n + 6$ "*

*A1 for " $6(n + 1)$ " or stating there is a factor of 6 oe*

*SC : B1 for  $n + n + 2 + n + 4$*

[5]

2.  $2 +$  'prime number' is odd

2

*M1 for a counter example showing intent to add 2 and another prime number; ignore incorrect examples*

*A1 for a correctly evaluated counter example with no examples given that involve either non-primes or incorrect evaluation*

**Alternative method**

*B2 for fully correct explanation '2 is a prime number, odd + even (or 2) = odd' oe with no accompanying incorrect statements or examples*

*(B1 for '2 is a prime number' or recognition that not all prime numbers are odd or odd + even (or 2) = odd; ignore incorrect examples or statements)*

[2]

3.  $(9n^2 + 6n + 1) - (9n^2 - 6n + 1) = 12n$   
 correct comment

3

*M1 for  $(3n)^2 + 3n + 3n + 1$  or  $(3n)^2 - 3n - 3n + 1$  or  $((3n + 1) - (3n - 1))((3n + 1) + (3n - 1))$*

*A1 for  $12n$  from correct expansion of both brackets*

*A1 for  $12n$  is a multiple of 4 or  $12n = 3 \times 4n$  or*

$$12n = 4 \times 3n \text{ or } \frac{12n}{4} = 3n \text{ or } \frac{12n}{3} = 4n$$

*NB: Trials using different values for  $n$  score no marks.*

[3]

1. There was a mixed response to part (a) with many variations of the correct answer. Part (b) was answered well but some pupils wrote  $4n + 2$  instead of  $2n + 4$ . Very few pupils gained marks on part (c). Candidates did not follow the lead given in the earlier parts of the question. Many gave a purely numerical answer and  $n + (n + 2) + (n + 4)$  was frequently seen.

2. Many candidates thought that 1 was a prime number. Others had trouble with the word “sum”, misinterpreting it as product.

Successful candidates usually offered a correct counter example, frequently  $2 + 3 = 5$ , and often backed this up by a written explanation. On occasions, a correct counter-example worthy of full marks was spoiled by further embellishment including incorrect statements or other examples involving non-primes.

3. Many candidates struggled with the requirement for an algebraic proof and instead opted to substitute various values for  $n$ . Those attempting to simplify the expression often made errors with  $(3n)^2$ , expressing it as  $9n$ ,  $6n^2$  or  $3n^2$ . Sign errors and omission of brackets around the second half of the expansions accounted for many of the other errors with  $1 \times 1 = 2$  causing a severe loss of marks for a few. A difference of two squares method was seen on a small number of occasions. Some candidates correctly simplified to  $12n$  but failed to justify the final mark often stating that 12 rather than  $12n$  was a multiple of 4.